METHOD OF CALCULATING THE ENERGY FLUX-DENSITY DISTRIBUTION IN AN IMAGE SPOT FOR A HEMISPHERICAL DIFFUSE-REFLECTION DEVICE

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A method of calculating the energy flux-density distribution in the image spot for a hemispherical diffuse-reflection device is proposed. The case of a diffusely emitting spot is specially considered.

In view of the extensive use of infrared technology in industry the need has arisen for a clear knowledge of various thermal-radiation characteristics of the materials employed. Special difficulties arise when measuring the spectral hemispherical transmission and reflection coefficients of materials T_λ and R_λ. For measuring directional hemispherical reflection and transmission coefficients, an attachment with a reflecting hemisphere is often employed [1-4].

The use of such an attachment fails to provide absolutely accurate values of the coefficients R_λ and T_λ. The main causes of error are as follows: 1) losses of radiation reflected from the sample through the entrance aperture in the hemisphere; 2) the screening action of the radiation-receiver mounting; 3) aberrations of the optical system, as a result of which the image of the emitting spot may exceed the dimensions of the receiving area on the receiver.

The losses of reflected radiation through the aperture in the hemisphere were considered in [3] for the case of diffuse reflection from the sample. In the same paper consideration was given to estimating the error due to optical aberrations, and it was concluded that the aberrations were minimized for the condition $s \leq 0.1R$.

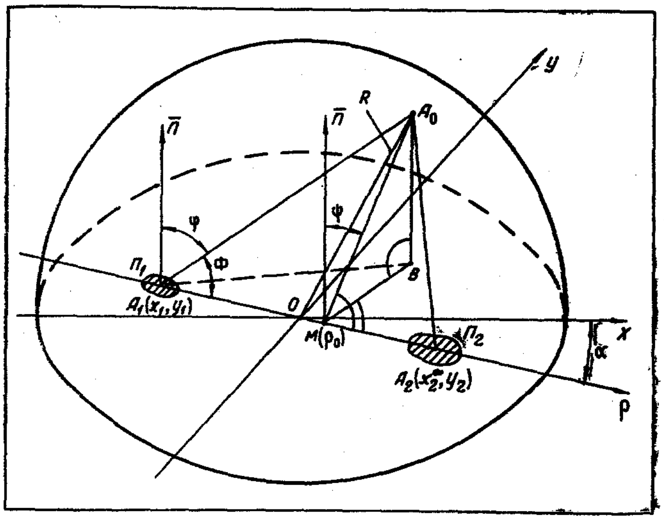

Fig. 1. Course of the rays in the system.

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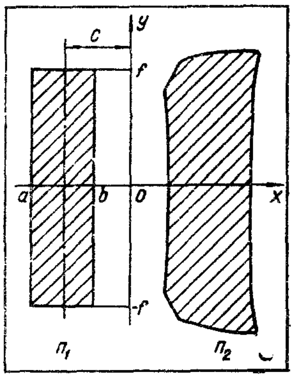

Fig. 2. Position and shape of the radiation spot in the Cartesian coordinate system XOY.

On the basis of this investigation by itself, the requirements to be imposed upon the radiation receiver cannot be formulated exactly. In the present investigation we therefore set ourselves the problem of determining the energy flux density in the image spot, so as to make a correct choice of the dimensions of the receiving area in the receiver, as well as its position, and also to calculate the measuring errors arising from optical aberrations. However, the energy flux-density distribution depends not only on the geometry, but also on the reflection indicatrix, of the sample. In the present instance we therefore considered the case of absolutely diffuse reflection.

Formulation of the Problem

Let us consider a concave reflecting hemisphere of radius R set on the coordinate plane XOY with the origin of coordinates at the center of the hemisphere (Fig. 1).

On the XOY plane under the hemisphere is a certain emitting spot Π_1 with a radiant energy density $E_1(x_1, y_1)$. The set of all the rays arising from Π_1 is reflected in the hemispherical mirror and creates an image spot Π_2 on the XOY plane.

We made the following assumptions: 1) The reflection coefficient of the reflecting hemisphere is equal to unity; 2) Π_1 emits diffusely at every point; 3) the absorption in the spot Π_2 is total; 4) the emitting spot Π_1 is arranged in such a way that for all its points, the condition $x_1 < 0$ is satisfied.

An additional requirement imposed upon the position of the spot Π_1 lies in the fact that every ray emerging from Π_1 is reflected once and only once in the hemispherical mirror before traveling to the image spot Π_2.

We note that the image of the point $A_1(x_1, y_1)$ is blurred so as to form a section of straight line, which passes through the points $A_1(x_1, y_1)$ and O, while the radiation arriving at the point $A_2(x_2, y_2)$ arises solely from those points which lie on the continuation of the straight line $OA_2(x_2, y_2)$ inside the spot Π_1.

Course of the Rays in the System

Let us consider that a ray leaves a point $A_1(x_1, y_1)$ of the emitting spot in a certain direction. Then the point $A_2(x_2, y_2)$ is that point of the spot Π_2 struck by this ray after being reflected at the point A_0 on the concave hemispherical mirror. The points $A_1(x_1, y_1)$ and $A_2(x_2, y_2)$ lie on a single diameter and on different sides of the center of the hemisphere. It follows from assumption, 3) that for all points of the spot $\Pi_2, x_2 > 0$.

Let us introduce a new coordinate axis ρ passing through the points O, $A_1(x_1, y_1)$, and $A_2(x_2, y_2)$. Let the values of ρ increase from point $A_1(x_1, y_1)$ to $A_2(x_2, y_2)$. The point O is taken as origin of coordinates. Thus, for all points belonging to Π_1 we have the condition $\rho_1 < 0$, while for the points in the image spot $\rho_2 > 0$. Since the hemisphere is symmetrical with respect to the ρ axis, we confine our attention to the plane case in our subsequent solution of the geometrical problem.

It follows from Fig. 1 that

$$A_0A_1 = \sqrt{R^2 - \rho_0^2 + (\rho_1 - \rho_0)^2}, \tag{1}$$

$$A_0A_2 = \sqrt{R^2 - \rho_0^2 + (\rho_2 - \rho_0)^2}. \tag{2}$$

The relationship between the coordinates of a point in the Cartesian coordinate system XOY and in the coordinate system $(\rho\alpha)$ is described by the expressions

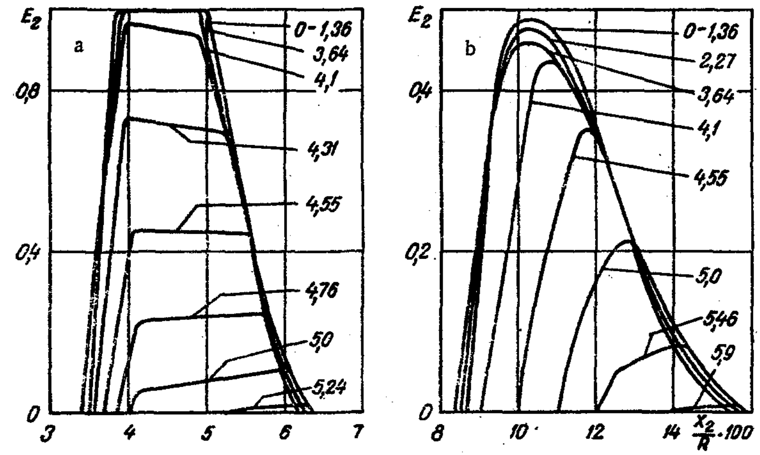

Fig. 3. Energy flux-density distribution in the image spot for a rectangular emitting spot: a) $a/R = \mp 0.0364$; $b/R = -0.0546$; $f/R = \div 0.0454$; $C/R = 0.0454$; b) $a/R = \div 0.1$; $b/R = \mp 0.1182$; $f/R = \mp 0.0454$; $C/R = 0.1091$ [the figures on the curves give the values of 100 (Y_2/R)].

$$\rho = \sqrt{x^2 + y^2}, \quad \alpha = \operatorname{arctg}\left|\frac{y}{x}\right|.$$

Since the angle of incidence is equal to the angle of reflection, it follows from the triangle $A_1A_0A_2$ that

$$\frac{A_0A_1}{A_0A_2} = \left|\frac{\rho_1}{\rho_2}\right|. \tag{3}$$

Using Eqs. (1), (2), and (3) we obtain a relationship between the coordinates of the three points A_0, A_1, and A_2:

$$\rho_0 = \frac{R^2(\rho_1 + \rho_2)}{2\rho_1\rho_2}. \tag{4}$$

On varying the coordinate ρ_0 from $+R$ to $-R$, the coordinate ρ_2 increases monotonically and at a certain moment reaches the value $\rho_2 = R$, which is equivalent to the appearance of a double reflection. From this we obtain the necessary (and sufficient) condition that the reflection from the hemisphere should be single for the points in the spot Π_1

$$\rho_1 \geqslant -\frac{R}{3}. \tag{5}$$

Let a ray having a direction defined by the angles φ and Φ (Fig. 1) leave the point $A_1(x_1, y_1)$ of the emitting spot. Using Fig. 1 we then have

$$\sin\Phi = \frac{A_0M}{A_0A_1} = \frac{\sqrt{R^2 - \rho_0^2}}{\sqrt{R^2 - \rho_0^2 + (\rho_1 - \rho_0)^2}}. \tag{6}$$

From triangles A_1A_0B, A_0MB and A_0A_1M we have

$$\cos\varphi = \sin\Phi\cos\psi. \tag{7}$$

Energy Density in the Image Spot

Let us consider the emitting spot Π_1. Let an elementary energy flux dq be emitted from a certain elementary area dF_1 of the diffusely emitting spot Π_1 with an emitted energy density $E_1(\rho_1, \alpha)$ in a direction making an angle φ with the normal to the XOY plane in an elementary solid angle $d\omega$ [5]:

$$dq = \frac{1}{\pi} E_1(\rho_1, \alpha) \cos\varphi dF_1 d\omega. \quad (8)$$

Clearly, the whole energy flux is focused by the hemisphere on an elementary area of the image spot dF_2. In the $(\rho\alpha)$ coordinate system the elementary areas may be expressed in the form

$$dF_1 = \rho_1 d\rho_1 d\alpha, \qquad dF_2 = \rho_2 d\rho_2 d\alpha.$$

The energy density in the spot Π_2 may then be written in the form

$$dE_2(\rho_2, \alpha) = \frac{1}{\pi} E_1(\rho_1, \alpha) \cos\varphi \sin\Phi \frac{\rho_1 d\rho_1}{\rho_2 d\rho_2} d\psi d\Phi. \quad (9)$$

Using Eqs. (6) and (4) we obtain

$$d\Phi = \frac{R^2 - \rho_0\rho_1}{\sqrt{R^2 - \rho_0^2}(R^2 - 2\rho_0\rho_1 + \rho_1^2)} d\rho_0, \quad (10)$$

$$d\rho_2 = \frac{2d\rho_0}{\left(\frac{2\rho_0}{R^2} - \frac{1}{\rho_1}\right)^2 R^2}. \quad (11)$$

It follows from Eqs. (7), (9), (10), and (11) that

$$dE_2(\rho_2, \alpha) = \frac{E_1(\rho_1, \alpha)\sqrt{R^2 - \rho_0^2}(R^2 - \rho_0\rho_1)(2\rho_0\rho_1 - R^2)^2}{2\pi(R^2 - 2\rho_0\rho_1 + \rho_1^2)^2 \rho_1\rho_2 R^2} \cos\psi d\psi d\rho_1. \quad (12)$$

Expressing ρ_0 in terms ρ_1 and ρ_2 and substituting into Eq. (12), we have

$$dE_2(\rho_2, \alpha) = \frac{E_1(\rho_1, \alpha) R^5 (\rho_2 - \rho_1)\sqrt{4\rho_1^2\rho_2^2 - R^2(\rho_1 + \rho_2)^2}}{8\pi(R^2 - \rho_1\rho_2)^2 \rho_1^2 \rho_2^3} \cos\psi d\psi d\rho_1. \quad (13)$$

Integrating with respect to ψ from $-\pi/2$ to $+\pi/2$ and with respect to ρ_1 from $\gamma(\rho_2, \alpha)$ to $\beta(\rho_2, \alpha)$, we obtain a final expression for $E_2(\rho_2, \alpha)$

$$E_2(\rho_2, \alpha) = \frac{1}{4\pi} \int_{\gamma(\rho_2, \alpha)}^{\beta(\rho_2, \alpha)} E_1(\rho_1, \alpha) \frac{R^5(\rho_2 - \rho_1)\sqrt{4\rho_1^2\rho_2^2 - R^2(\rho_1 + \rho_2)^2}}{(R^2 - \rho_1\rho_2)^2 \rho_1^2\rho_2^3} d\rho_1. \quad (14)$$

Equation (14) may be reduced to dimensionless form

$$E_2(\rho_2', \alpha) = \frac{1}{4\pi} \int_{\gamma(\rho_2', \alpha)}^{\beta(\rho_2', \alpha)} E_1(\rho_1', \alpha) \frac{(\rho_2' - \rho_1')\sqrt{4(\rho_1')^2(\rho_2')^2 - (\rho_1' + \rho_2')^2}}{(1 - \rho_1'\rho_2')^2(\rho_1')^2(\rho_2')^3} d\rho_1', \quad (15)$$

where $\rho' = \rho/R$.

We see from Eqs. (14) and (15) that the energy flux-density distribution at the point $A_2(x_2, y_2)$ of the image spot is calculated by summing the energy contributions introduced by emitting points lying on the continuation of the straight line joining point $A_2(x_2, y_2)$ to the center of the hemisphere, i.e., the ρ axis.

The limits of integration $\gamma(\rho_2, \alpha)$ and $\beta(\rho_2, \alpha)$ are determined by the geometrical dimensions of the emitting spot and by the conditions arising from Eqs. (4) and (5). It may readily be shown that on the basis of condition (5), the range of variation of ρ_2 is divided into two parts. For $0 < \rho_2 \leq R/5$ the range of variation of ρ_1 is given by the expression

$$\frac{1}{\frac{2}{R} - \frac{1}{\rho_2}} \leqslant \rho_1 \leqslant \frac{1}{-\frac{2}{R} - \frac{1}{\rho_2}}, \quad (16)$$

while for $R/5 < \rho_2 < R$ it appears as follows

$$-\frac{R}{3} \leqslant \rho_1 \leqslant \frac{1}{-\frac{2}{R} - \frac{1}{\rho_2}}. \quad (17)$$

In order to find $\gamma(\rho_2, \alpha)$ and $\beta(\rho_2, \alpha)$ for the point $A_2(x_2, y_2)$, we must simultaneously consider both the geometrical ranges of variation of ρ_1 (coordinates of the points at which the ρ axis intersects the boundary of the emitting spot) and also Eqs. (16) and (17).

The foregoing method of calculation was realized in the form of a program for the Minsk-22 computer. The energy flux density was calculated in the image spot Π_2 for a rectangular emitting spot Π_1 and a number of values of C/R and $E_1(\rho_1, \alpha)=1$ (Figs. 2 and 3).

Thus, for the case of diffuse reflection we may use the foregoing method of calculating the energy flux density in the image spot to determine the measuring error due to optical aberrations very accurately and subsequently to reduce it to a minimum.

NOTATION

R, radius of reflecting hemisphere; α, angle between the positive directions of the ρ and OX coordinate axes; $\bar{n}$, normal to the XOY plane; φ, angle between $\bar{n}$ and the direction in which the ray leaves the emitting spot; Φ, angle between the positive direction of the coordinate axis ρ and the direction in which the ray leaves the emitting spot; ψ, angle between $\bar{n}$ and a perpendicular dropped from point A_0 to the ρ axis; C, distance between the center of the hemisphere and the geometrical center of the rectangular emitting spot; a, b, and f, coordinates of the rectangular emitting spot. Indices 0, 1, 2, refer to the surface of the hemisphere, the emitting spot, and the image spot, respectively.

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